Christ Church Grammar School

Semester One Examination, 2018

Question/Answer booklet

MATHEMATICS METHODS UNIT 1

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Your name ______________________________

Teacher's name ______________________________

Time allowed for this section

Reading time before commencing work:	ten minutes
Working time:	one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper (both sides), and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	14	14	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

SN018-112-6

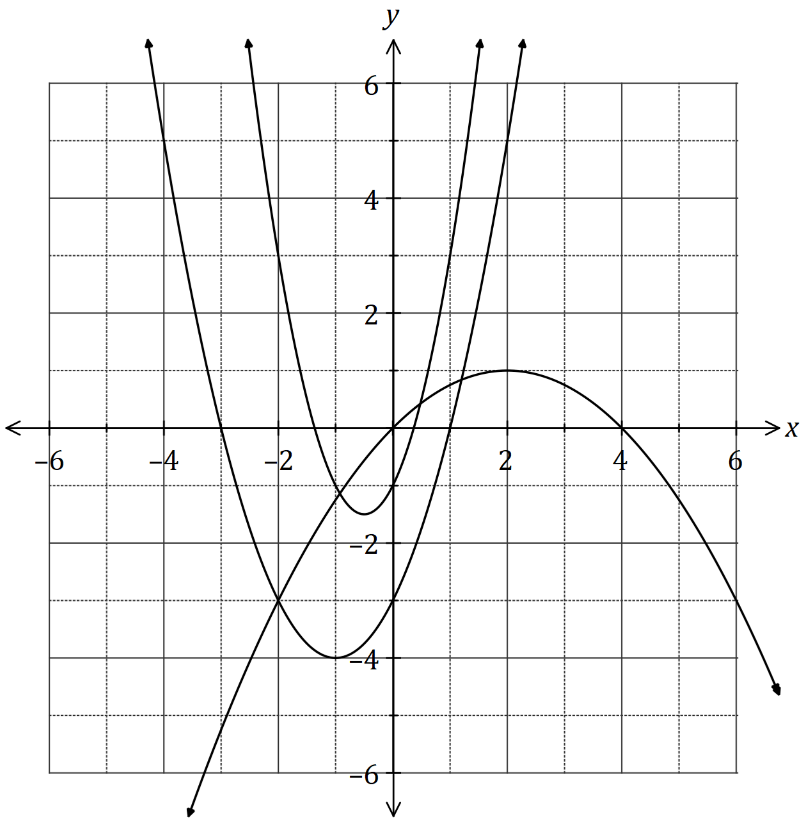
Section Two: Calculator-assumed **65% (98 Marks)**

This section has **fourteen (14)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(3 marks)**

The graphs of $y = 2x^2 + 2x + c$, $y = a(x-2)^2 + 1$ and $y = (x+b)(x+3)$ are shown below.



Determine the values of the constants a, b and c.

Question 10 **(5 marks)**

An online grocery is offering new customers the opportunity to select 8 different products for just $2 each. They can select from a range of 12 different canned items, 14 different snacks and 13 different drinks.

(a) Determine how many different selections can be made. (1 mark)

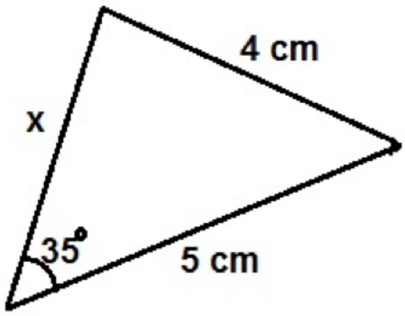
(b) Determine how many different selections can be made that just include drinks. (1 mark)

In a separate offer, the online grocery forms a special bargain pack containing 4 different canned items and 3 different snacks.

(c) How many different special bargain packs can be formed? (3 marks)

Question 11 **(3 marks)**

Calculate the value of the distance x in the following triangle.



Question 12 **(6 marks)**

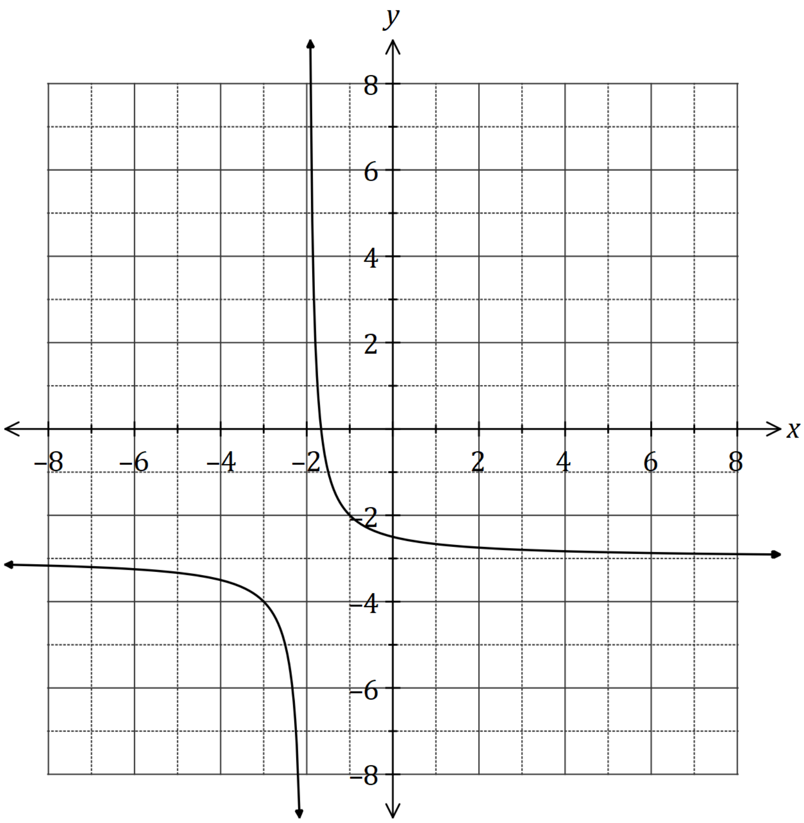
(a) Determine the equation of the axis of symmetry for the graph of $y = 3x^2 + 12x + 40$.
(2 marks)

(b) The graph of $y = ax^2 + bx + 13$ passes through the points $(-3, -23)$ and $(4, 5)$.
Determine the values of the constants a and b. (4 marks)

Question 13 **(7 marks)**

Let $f(x) = \dfrac{4}{3-x}$ and $g(x) = \dfrac{1}{x+p} + q$, where p and q are constants.

The graph of $y = g(x)$ is shown below.



(a) Sketch the graph of $y = f(x)$ on the axes above. (3 marks)

(b) Determine the values of p and q. (2 marks)

(c) Solve the equation $f(x) = g(x)$, giving your solution(s) to one decimal place. (2 marks)

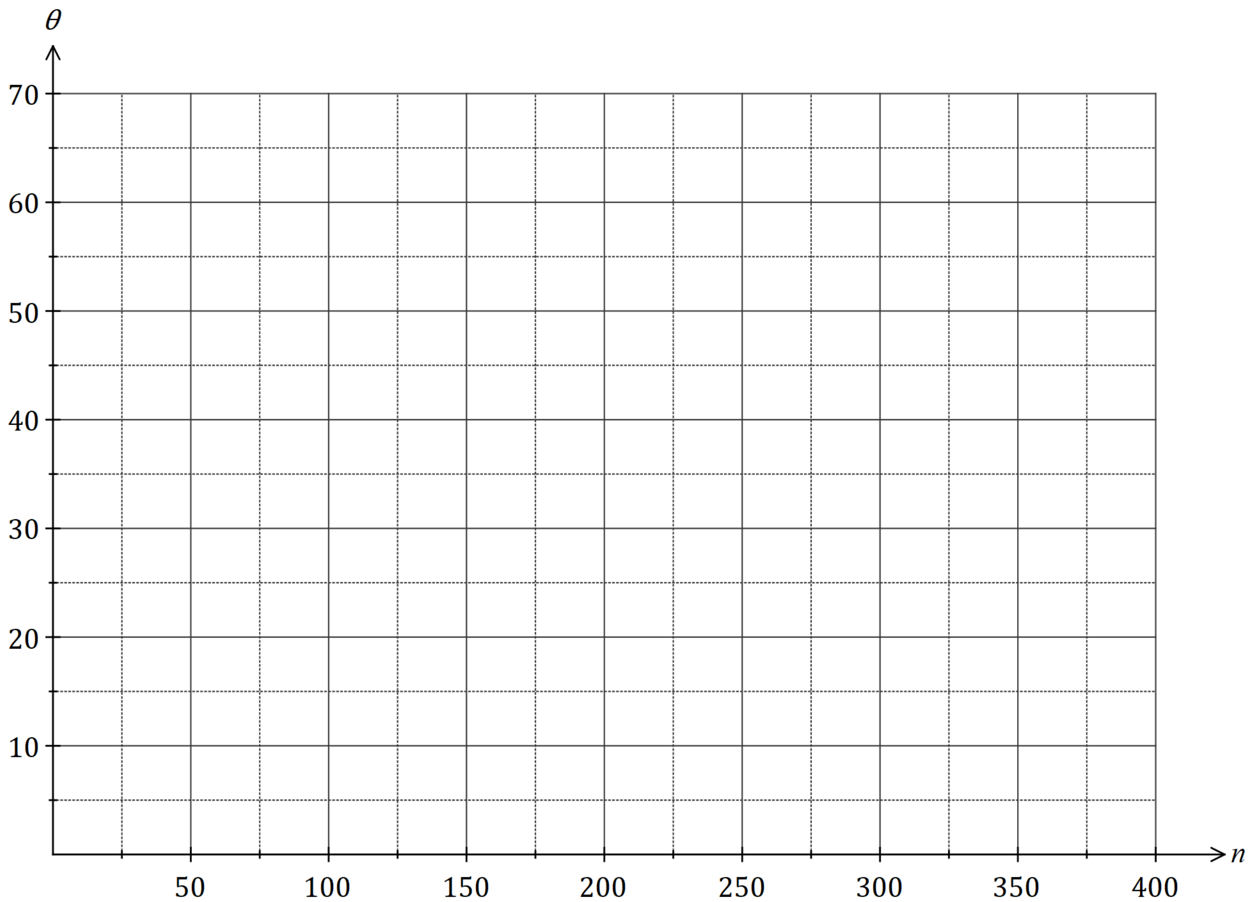
Question 14 **(12 marks)**

During 2018, the altitude of the sun, θ degrees, at noon in Paris on the n^{th} day of the year can be modelled by the equation

$$\theta = 23.5\sin\left(\frac{8\pi(n + 283)}{1461}\right) + 41.1$$

(a) On the 30th of January, the altitude was $22.7°$. Calculate the altitude twelve days later. (2 marks)

(b) Graph the altitude on the axes below for $0 \le n \le 365$. (4 marks)



Question 14 (continued)

(c) State the maximum altitude of the sun at noon in Paris and on which day of the year this occurred. (2 marks)

(d) Solar panels on the roof of a Paris apartment are designed to meet its entire power needs on cloudless days when the altitude of the sun is at least $30°$ at noon.

(i) Draw a straight line on the axes grid in part (b) to represent this requirement. (1 mark)

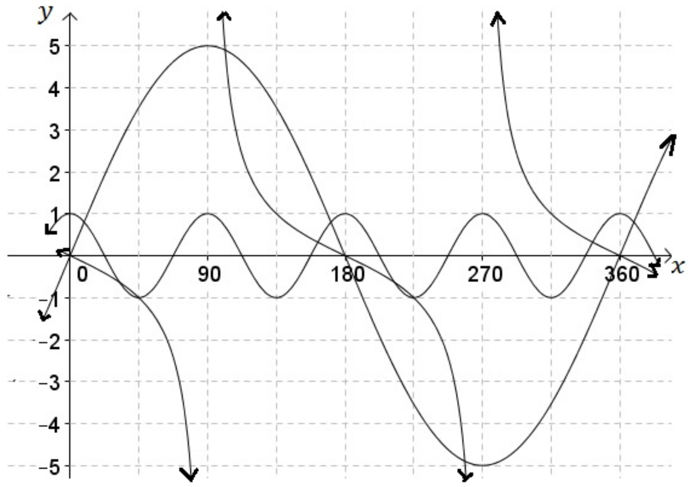
(ii) Determine the number of days the panels are not expected to achieve this aim during 2018, ignoring the possibility of cloud cover. (3 marks)

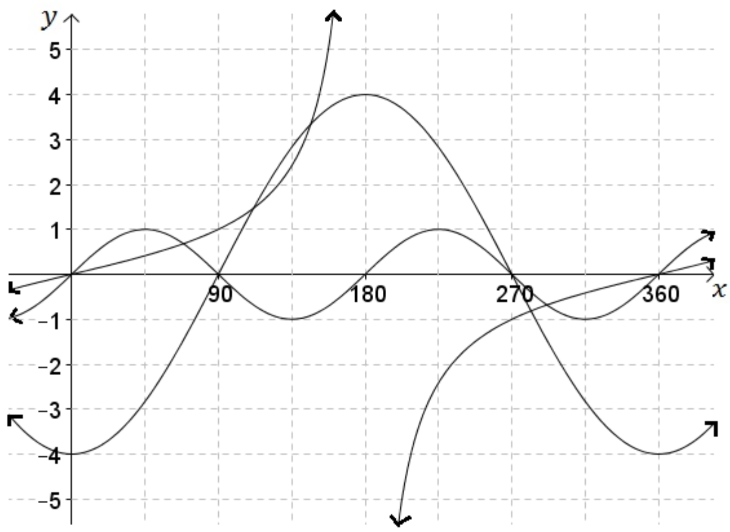
Question 15 **(9 marks)**

(a) The graphs of the following, where a, b, c, d, e and f are constants, are shown in the two axes grids below.

$y = \sin(ax)$ $\quad y = b\cos(x)$ $\quad y = \tan(cx)$ $\quad y = d\sin(x)$ $\quad y = \cos(ex)$ $\quad y = f\tan(x)$

Three of the graphs are in the first axes grid and three of the graphs are in the second axes grid.





State the values of a, b, c, d, e and f. (6 marks)

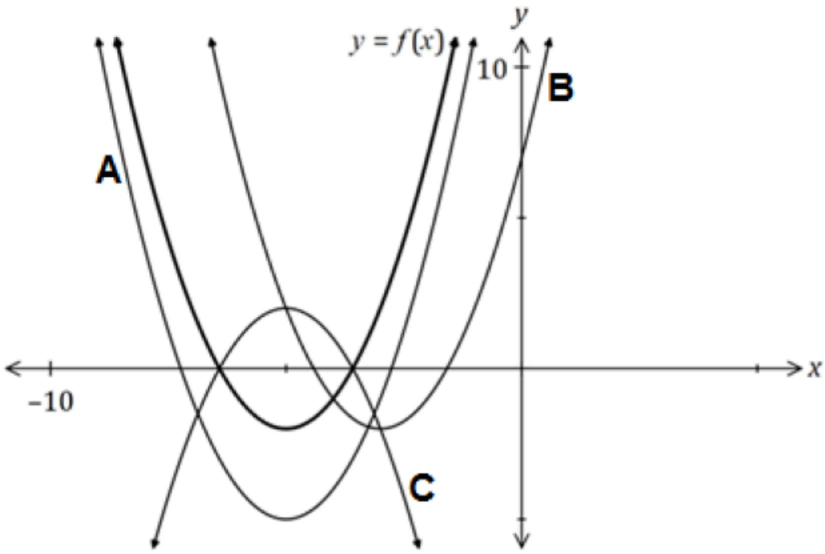
Constant	Value
a	
b	
c	
d	
e	
f	

Question 15 (continued)

(b) Calculate the acute angle in degrees between the lines $y = x + 5$ and $y = 3x - 1$, rounding your answer to one decimal place. (3 marks)

Question 16 **(6 marks)**

(a) The graph of $y = f(x)$ is shown in bold below. The graphs of $y = -f(x)$, $y = f(x+p)$ and $y = f(x) + q$ are also shown, where p and q are constants.



Complete the table below giving the equation of each of the graphs A, B and C as one of $y = -f(x), y = f(x+p)$ or $y = f(x) + q$.

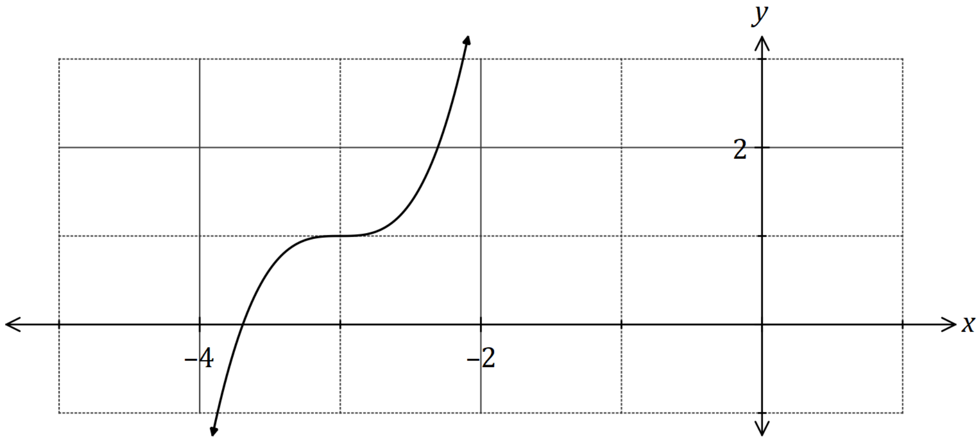
Graph	Equation
A	
B	
C	

(3 marks)

(b) The one-to-one relation $y = 9 - 2x$ has domain and range given by $\{x : x = -4, a, 10\}$ and $\{y : y = -11, -7, b\}$ respectively. Determine the values of constants a and b. (3 marks)

Question 17 **(6 marks)**

(a) Part of the graph of $y = f(x)$ is shown below, where $f(x) = 3(x + b)^3 + c$, and b and c are constants.



(i) State the degree of $f(x)$. (1 mark)

(ii) Determine the value of b. (1 mark)

(iii) Determine $f(0)$. (2 marks)

(b) Another function, $g(x)$, is a transformation of $f(x)$, where $g(x) = f(x - 7)$.

Describe how to obtain the graph of $y = g(x)$ from the graph of $y = f(x)$. (2 marks)

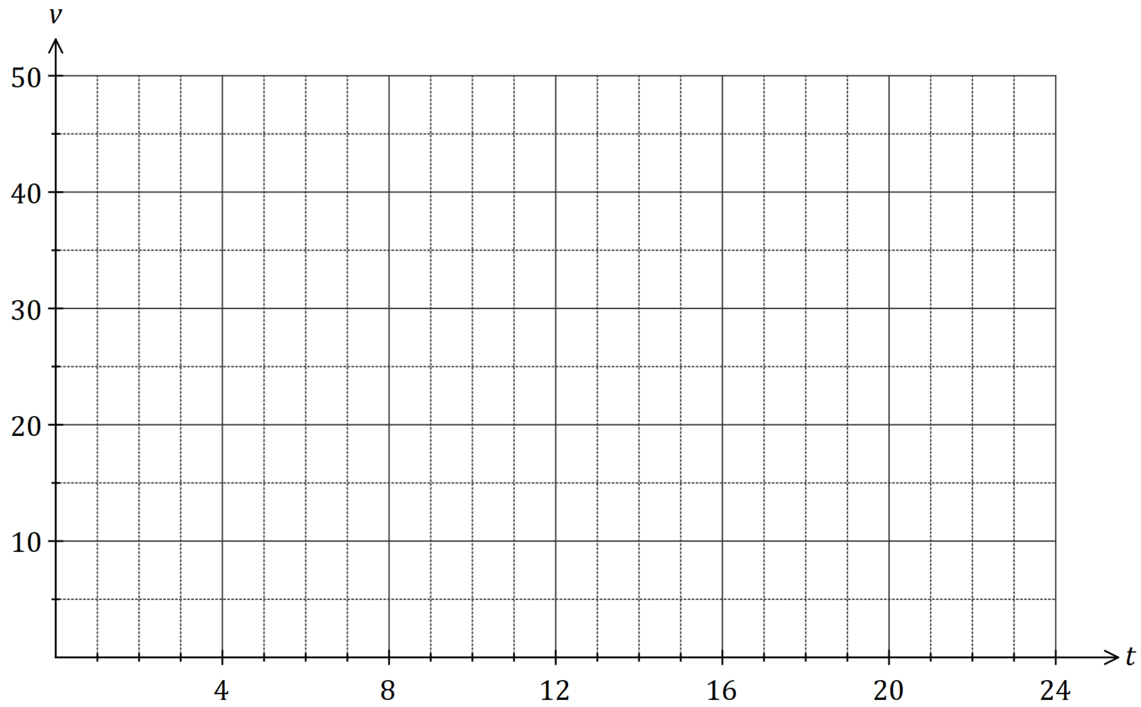
Question 18 **(9 marks)**

The wind speed at a weather station, v metres per second, t hours after recording began, can be modelled by the function

$$v = 20 - 5.8t + 0.75t^2 - 0.02t^3, 0 \leq t \leq 24$$

(a) Calculate the wind speed when $t = 11$. (1 mark)

(b) Sketch the graph of wind speed against time on the axes below. (4 marks)



(c) During the 24-hour period, determine

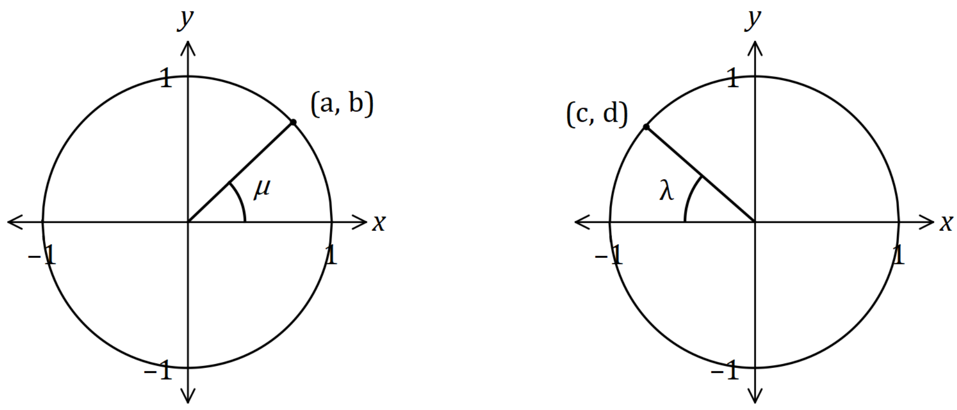
(i) the time at which the wind speed was greatest. (1 mark)

(ii) the minimum wind speed. (1 mark)

(iii) the length of time, in hours and minutes, that the wind speed was increasing. (2 marks)

Question 19 **(7 marks)**

Consider the points with coordinates (a, b) and (c, d) that lie in the first and second quadrants respectively of the unit circles shown below, where μ and λ are acute angles.



Determine the following in terms of a, b, c and d, simplifying your answers where possible.

(a) $\cos \lambda$. (1 mark)

(b) $\tan(180° + \mu)$. (1 mark)

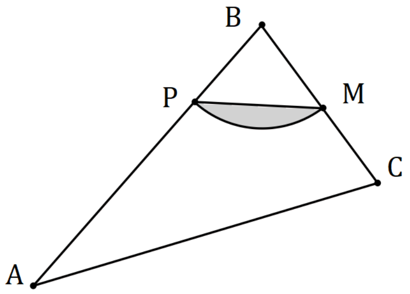
(c) $\sin(\pi + \lambda)$. (1 mark)

(d) $\cos(90° - \mu)$. (1 mark)

(e) $\sin(\mu - \lambda)$. (3 marks)

Question 20 **(10 marks)**

A logo with triangular outline ABC contains a shaded segment bounded by the straight line PM and the circular arc PM with centre B and radius $BM = 32$ cm, as shown below.



Given that $\angle ABC = \frac{4\pi}{9}$, $\angle BCA = 3\angle BAC$ and M is the midpoint of BC, determine

(a) the size of $\angle ABC$ in degrees. (1 mark)

(b) the area of the shaded segment. (2 marks)

(c) the perimeter of the shaded segment. (3 marks)

Question 20 (continued)

(d) the area of triangle ABC. (4 marks)

Question 21 **(7 marks)**

Consider the equation

$$2kx^2 - 4x + k = 0$$

(a) Solve this equation for x, giving your answer as a simplified expression in terms of k. (3 marks)

(b) Give the value(s) of k for which the equation has exactly one solution. (2 marks)

Question 21 (continued)

(c) Calculate the value(s) of $\boldsymbol{x}$ when $\boldsymbol{k}$ = *-1.2.* (2 marks)

Question 22 **(8 marks)**

(a) Use your calculator to

(i) determine the exact value of $\cos 36°$. (1 mark)

(ii) determine the exact value of $\sin 105°$. (1 mark)

(iii) solve $\cos(x + 60°) = \sin x$ for $-270° \leq x \leq 270°$. (1 mark)

(b) Using suitable exact values of acute angles and an angle sum and difference identity, justify your above value of $\sin 105°$. (5 marks)

End of questions

Additional working space

Question number: _________

Additional working space

Question number: _________

Additional working space

Question number: _________